Savitribai Phule Pune University

(Formerly Known as University of Pune)

Revised Syllabi

of

T.Y. B.A. Statistics (Special &General)
(2013 Pattern)

To be effective from

Academic Year 2015-2016

Papers offered at TYBA Statistics (Special &General level)

STATISTICS (GENERAL) : 1 Paper

STATISTICS (SPECIAL) : 2 Papers (1 Theory, 1 Practical)

MATHEMATICAL STATISTICS (GENERAL) : 1 Paper

APPLIED STATISTICS (GENERAL) : 1 Paper

STATISTICAL PRE-REQUISITES (GENERAL) : 1 Paper

Equivalence of old papers (2010-11 to 2014-15) with new papers (2015 onwards) in TYBA statistics (Special &General)

Papers in old course **(2010-11 to 2014-15)**	**Papers in new course** **(2015 onwards)**	**Papers in new course** **(2015 onwards)**
Statistics (General) Title: Design of Experiments & Operations Research	Statistics (General) Title: Design of Experiments & Operations Research	No equivalence
Statistics (Special - III) Title: Distribution Theory	Statistics (Special - III) Title: Distribution Theory	No equivalence
Statistics (Special - IV) Title: Practicals	Statistics (Special - IV) Title: Practicals	No equivalence
Mathematical Statistics(General) Title: Statistical Inference	Mathematical Statistics (General) Title: Statistical Inference	Equivalent Paper
Applied Statistics (General) Title: Applications of Statistics	Applied Statistics (General)	Equivalent Paper
Statistical Prerequisites (General)	Statistical Prerequisites (General)	No equivalence

STATISTICS (General and Special)

Note: 1) A student of the Three-Year B.A. Degree Course offering 'Statistics' at the special level must offer `Mathematical Statistics' as a General level subject in all the three years of the course. Further students of the three-year B.A. Degree course are advised not to opt 'Statistics' as the general level subject unless they have offered 'Mathematical Statistics' as a General level subject in all the three years of the course.

2) A student of three year B..A. degree course opting for 'Statistics' will not be allowed to opt 'Applied Statistics' in any of the three years of the course.

3) A student opting `Statistics' at the Special level must complete all practicals in Practical Paper to the satisfaction of the teacher concerned.

4) He/ She must produce the laboratory journal along with the completion certificate signed by the Head of the Department at the time of Practical Examination.

5) **Structure of evaluation of practical paper at T.Y.B.A**

(A) Continuous Internal Evaluation		**Marks**	
(i)	Journal	10	
(ii)	Viva-voce	10	
Total(A)		**20**	
(B) Annual practical examination			
Section	Nature	Marks	Time
I	Attempt **any two** out of **four questions** each of 35 marks :	70	3 hours on calculator 2 hours 30 minutes on computer
II	Viva-voce	10	10 minutes
	Total(B)	**80**	3 Hours and 10 minutes
	Total (A) + (B)	**100**	---

6) Duration of the practical examination be extended by 10 minutes to compensate for the loss of time for viva-voce of the candidates.

7) Batch size should be of maximum 12 students.

8) To perform and complete the practicals, it is necessary to have computing facility. So there should be sufficient number of computers, UPS and electronic calculator in the laboratory.

9) In order to acquaint the students with applications of statistical methods in various fields such as industries, agricultural sectors, government institutes, etc. at least one Study Tour for T.Y. B.A. Statistics students must be arranged.

10) Examples and Problems in theory papers should be covered where ever these are necessary.

STATISTICS (GENERAL)

Design of Experiments and Operations Research

First Term

DESIGN OF EXPERIMENTS

1. Design of Experiments (24 L)

1.1 Analysis of variance (ANOVA): concept and technique.

1.2 Basic terms of design of experiments: Experimental unit, treatment, layout of an experiment.

1.3 Basic principles of design of experiments : Replication, randomization and local control. Choice of size and shape of a plot for uniformity trials, the empirical formula for the variance per unit area of plots.

1.4 Completely Randomized Design (CRD) : Application of the principles of design of experiment in CRD, Layout, Model:

$X_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ i= 1,2, …….t. j = 1,2,………., n_i

Assumptions and interpretations. Testing normality graphically. Breakup of total sum of squares into components. Estimation of parameters, expected values of mean sums of squares, components of variance, preparation of (ANOVA) table, hypothesis to be tested

$H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_t = 0$. Comparison of treatment means using box plot techniques. Statement of Cochran's theorem. F test for testing H_0 with justification (independence of chi- square is to be assumed), testing for equality of two specified treatment effects, comparison of treatment effects using critical difference.

1.5 Randomized Block Design (RBD) : Application of the principles of design of experiments in RBD, layout model:

$X_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$ i= 1,2, …….t. j = 1,2,………. b, Assumptions and interpretations.

Breakup of total sum of squares into components. Estimation of parameters, expected values of mean sums of squares, components of variance, preparation of analysis of variance table, hypotheses to be tested

$H_{01} : \alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_t = 0$

$H_{02} : \beta_1 = \beta_2 = \beta_3 = \ldots = \beta_b = 0$

F test for testing H_{01} and H_{02} with justification (independence of chi- squares is to be assumed), testing for equality of two specified treatment effects, comparison of treatment effects using critical difference,

1.6 Latin Square Design (LSD): Application of the principles of design of experiments in LSD, layout, Model :

$$X_{ij(k)} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ij(k)} \quad i = 1,2, \cdots m, \quad j = 1,2\cdots,m, \ k=1,2\cdots,m.$$

Assumptions and interpretations.

Breakup of total sum of squares into components. Estimation of parameters, expected values of mean sums of squares, components of variance , preparation of analysis of variance table, hypotheses to be tested.

$H_{01} : \alpha_1 = \alpha_2 = \cdots = \alpha_m = 0$

$H_{02} : \beta_1 = \beta_2 = \cdots = \beta_m = 0$

$H_{03} : \gamma_1 = \gamma_2 = \cdots = \gamma_m = 0$

F test for H_{01}, H_{02} and H_{03} with Justification (independence of chi- square is to be assumed). Preparation of ANOVA table and F test for H_{01}, H_{02} and H_{03}, testing for equality of two specified treatment effects, comparison of treatment effects using critical difference.

1.7 Linear treatment contrasts, orthogonal contrasts. Scheffe's method for comparing contrasts, Tuckey's procedure for comparing pairs of treatment means(applicable to C.R.D., R.B.D. and L.S.D.)

1.8 Identification of real life situations where the above designs are used.

1.9 Analysis of non- normal data using.

i) Square root transformation for counts.

ii) $Sin^{-1}(.)$ transformation for proportions.

iii) Kruskal Wallis test.

2. Efficiency of Design **(5L)**

2.1 Concept and definition of efficiency of a design.

2.2 Efficiency of RBD over CRD.

2.3 Efficiency of LSD over (i) CRD (ii) RBD.

3. Analysis of Covariance (ANOCOVA) with One Concomitant Variable **(7L)**

3.1 Situations where analysis of covariance is applicable.

3.2 Model for covariance in CRD, RBD. Estimation of parameters (derivations are not expected)

3.3 Preparation of analysis of variance – covariance table, test for $\beta=0$, test for equality of treatment effects (computational technique only).

4. Factorial Experiments **(12L)**

4.1 General description of m^n factorial experiment, 2^2 and 2^3 factorial experiments arranged in RBD.

4.2 Definitions of main effects and interaction effects in 2^2 and 2^3 factorial experiments.

4.3 Yate's procedure, preparation of ANOVA table, test for main effects and interaction effects.

4.4 General idea of confounding in factorial experiments.

4.5 Construction of layouts in total confounding and partial confounding in 2^2 and 2^3 factorial experiments.

4.6 Total confounding (confounding only one interaction) ANOVA table, testing main effects and interaction effects.

4.7 Partial confounding (confounding only one interaction per replicate); ANOVA table, testing main effects and interaction effects.

Second Term

OPERATIONS RESEARCH

5. Linear Programming **(16 L)**

5.1 Statement of the linear Programming Problem (LPP),(minimization and maximization) Formulation of problem as L.P. problem.
Definition of (i) A slack variable, (ii) A surplus Variable.

L.P. Problem in (i) Canonical form ,(ii) standard form.
Definition of (i) a solution (ii)basic and non basic variables (iii) a feasible solution(iv) a basic feasible solution, (v) a degenerate and non–degenerate solution (vi) an optimal solution.

5.2 Solution of L.P.P by Simplex Method: Obtaining Initial Basic Feasible Solution (IBFS) , criteria for deciding whether obtained solution is optimal ,criteria for unbounded solution , no solution , more than one solutions , introduction of artificial variable, Big-M method.

5.3 Duality Theory: Writing dual of a primal problem, solution of a L.P.P. by using its dual problem.

6. Transportation Problem and Assignment Problem (12 L)

6.1 Transportation problem (T.P.): Statement of T.P., balanced and unbalanced T.P. Minimization and maximization problem.

6.2 Obtaining basic feasible solution of T.P. by (i) Least cost method (ii) Vogel's approximation method (VAM).

6.3 u-v (MODI) method of obtaining Optimal solution of T.P., uniqueness and non-uniqueness of optimal solutions, degenerate solution

6.4 Assignment Problem : Statement of an assignment problem , Minimization and maximization problem , balanced and unbalanced problem ,relation with transportation problem , optimal solution using Hungarian method , maximization case

7.Simulation (8 L)

7.1 Introduction to simulation, merits, demerits, limitations.

7.2 Pseudo random number generates: Linear congruential , mid square method.

7.3 Model sample from normal distribution (using Box- Muller transformation), uniform distribution, exponential distribution.

7.4 Monte Carlo method of simulation: Statistical applications of simulation in numerical integration such as computation of probabilities of events related to gamma , beta and bivariate normal distribution.

8. Critical Path Method (CPM) and Project Evaluation and Review Techniques (PERT) (12 L)

8.1 Definition of (i) Event,(ii) Node,(iii)Activity,(iv)Critical Activity,(v)Project Duration.

8.2 CPM: Construction of network, Definitions

(i) earliest start time
(ii) earliest finish time
(iii) latest start time
(iv) latest finish time for an activity.

Critical Path , Types of float , total floats, free float, independent float and their significance. Determination of critical path

8.3 PERT: Construction of network; (i) pessimistic time estimate, (ii) optimistic time estimate (iii) most likely time estimates, Determination of critical path, determination of mean and standard deviation of project duration, computations of probability of completing the project in a specified duration.

Books Recommended

1. Cochran W.G. and Cox, C.M. (1968) Experimental Design, John Wiley and Sons, Inc., New York.
2. Dass, M.N. and Giri,N.C. (1986) Design and Analysis of Experiments, II Edition Wiley Eastern Ltd., New Delhi
3. Federer W.T. (1967) Experimental Design : Oxford and IBH Publishing Co., New Delhi.
4. Gass, S.L. (1997). Linear programming methods and applications, Narosa Publishing House, New Delhi.
5. Goon, A.M., Gupta,M.K. and Dasgupta, B. (1998). Fundamentals of Statistics, Vol.II, The world Press Pvt. Ltd. Kolkatta.
6. Gupta S.C. and Kapoor V.K.(2006). Fundamentals of Applied Statistics, S.Chand Sons, New Delhi.
7. Gupta, P.K. and Hira, D.S.(2008). Operation Research, 3rd edition S. Chand and company Ltd., New Delhi.
8. Johnson, R.A., Miller, I. and Freund, J.(2010). Probability and Statistics for engineers, Prentice Hall, India.
9. Kapoor, V. K.(2006). Operations Research, S. Chand and Sons. New Delhi.
10. Kempthorne, O. (1952). Design of Experiments, Wiley Eastern Ltd., New Delhi.
11. Montgomery, D.C. (2001). Design and Analysis of Experiments, John Wiley and sons Inc., New Delhi.
12. Phillips, D.T and Solberg, R.A.(1976). Operation Research principles and practice, John Willey and sons Inc.
13. Saceini, M., Yaspan,A.. and Friedman, L.(2013). Operation Research methods and problems, Willey International Edition.
14. Sharma, J.K. (1989). Mathematical Models in Operation Research, Tata McGraw Hill Publishing Company Ltd., New Delhi.
15. Shrinath.L.S (1975). Linear Programming, Affiliated East-West Pvt. Ltd, New Delhi.
16. Snedecor, G.W. and Cochran, W.G. (1994). Statistical Methods, 8th edition, Affiliated East – West Press, New Delhi.

17.Taha, H.A. (2007). Operation research: An Introduction, eighth edition, Prentice Hall of India, New Delhi.

18.Wu,C.F.J. and Hamda, M. (2009). Experiments, Planning, Analysis and Parameter Design Optimization, John Wiley & Sons,Inc.,Hoboken,New Jersey.

STATISTICS (SPECIAL- III)

DISTRIBUTION THEORY

First Term

1. Beta distribution **(12L)**

1.1 **Beta distribution of first kind**

p.d.f. $f(x) = \frac{1}{B(m,n)} x^{m-1}(1-x)^{n-1}$, $0 \le x \le 1$, m ,n >0

= 0, elsewhere

Notation: $X \sim \beta_1(m,n)$

Nature of probability curve,

Derivation of mean, variance, r^{th} raw moment, harmonic mean,mode.

Symmetry of the distribution .

1.2 Relation with U (0, 1).

Probability distributions of

$\frac{1}{X}$, X+Y, X-Y, XY, $\frac{X}{Y}$, where X and Y are iid β_1 (1,1),

1.3 **Beta distribution of second kind**

p.d.f $f(x) = \frac{1}{B(m,n)} \cdot \frac{x^{m-1}}{(1+x)^{m+n}}$, $x \ge 0$, m ,n >0

= 0, elsewhere

Notation: $X \sim \beta_2(m,n)$

Nature of probability curve ,Derivation of mean, variance, r^{th} raw moment, harmonic mean,mode .

1.4 Derivation of interrelation between β_1 (m, n) and β_2 (m, n).

1.5 Derivation of distribution of $\frac{X}{Y}$, $\frac{X}{X+Y}$, when X and Y are independent gamma variates.

1.6 Statement of relation between distribution function of β_1 (m, n) and binomial distribution.

2. Weibull Distribution **(8L)**

2.1 p.d.f .

$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}$, $x \ge 0$, α , β >0

= 0, elsewhere

Notation : X~W (α , β) .

2.2 Probability curve, location parameter, shape parameter, scale parameter . Derivation of distribution function , quartiles ,mean and variance ,coefficient of variation , relationship with gamma and exponential distribution.

3. Laplace (Double Exponential) Distribution (8 L)

3.1 p.d.f. $f(x) = \frac{\lambda}{2}\exp(\ -\lambda\ |x - \mu|\ )$; $-\infty<x<\infty$, $-\infty< \mu <\infty$, $\lambda >0$,

$= 0$; elsewhere

Notation: X~ L (μ , λ)

3.2 Nature of the probability curve .

3.3 Derivation of distribution function, quartiles.

3.4 MGF, CGF, Moments and cumulants, skewness and kurtosis

3.5 Derivation of Laplace distribution as the distribution of the difference of two i.i.d. exponential random variables with mean $\frac{1}{\lambda}$.

4. Cauchy Distribution (10L)

4.1 p.d.f. $f(x) = \frac{\lambda}{\pi}\ \frac{1}{\lambda^2+(x-\mu)^2}$; $-\infty<x<\infty$, $-\infty< \mu <\infty$, $\lambda >0$.

Notation : X~ C (μ , λ)

4.2 Nature of the the probability curve, comparison with tails of normal distribution .

4.3 Derivation of distribution function, quartiles .Non – existence of moments, Statement of distribution of $aX + b$. Derivation of distribution of i) $\frac{1}{X}$ ii) X^2 where X~ C (0,1), Problems based on these results.

4.4 Statement of additive property for two Independent Cauchy variates, statement of distribution of the sample mean ,comment on limiting distribution of $\overline{X}$.

4.5 Statement of relationship with uniform, student's t and normal distributions.

5. Lognormal Distribution (10 L)

5.1 p.d.f $f(x) = \frac{1}{(x-a)\sigma\sqrt{2\pi}}\ \exp\left\{\frac{-1}{2\sigma}\ [\log_e(x-a)-\mu]^2\right\}$, $a<x$, $-\infty< \mu <\infty$, $\sigma >0$,

$= 0$; elsewhere

Notation: X~ LN (a ,μ , σ^2)

5.2 Derivation of relation with N (μ, σ^2) distribution

5.3 Nature of the probability curve.

5.4 Derivation of moments (r-th moment of X-a), mean, variance, quartile, mode, Karl Pearson's and Bowley's coefficient of skewness and kurtosis, derivation of quartiles and mode.

5.5 Distribution of (Π Xi), when Xi's independent lognormal random variables .

Second Term

6. Bivariate Normal Distribution. (14L)

6.1 p.d.f of a bivariate normal distribution.

$$f(x) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho 2}} \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 + \left(\frac{y-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right)\right]\right\}$$

$-\infty < x, y < \infty$,

$-\infty < \mu_1 , \mu_2 < \infty$,

$\sigma_1, \sigma_2 > 0, -1 < \rho < +1$

Notation: $(X, Y) \sim BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$,

6.2 Nature of surface of p. d. f., marginal and conditional distributions , identification of parameters , regression of Y on X, independence and uncorrelatedness, Derivation of MGF and moments . Statement of distribution of aX+ bY+ c , X/Y.

7. Order statistics (10 L)

7.1 Order Statistics for a random sample of size n from a continuous distribution, definition, derivation of distribution function and density function of the i^{th} order statistics $X_{(i)}$, particular cases for i = l and i = n.

7.2 Distribution of $X_{(i)}$ for a random sample from uniform and exponential distribution.

7.3 Definition of p-th sample quantile $X_{([np]+1)}$. Distribution of sample median for a random sample from uniform distribution.

8. Central Limit Theorem and Weak Law of Large Numbers (10 L)

8.1 Sequence of r.v.s., convergence of sequence of r.v. in a) probability b) distribution with simple illustrations.

8.2 Statement and proof of the central limit theorem for i.i.d. r.v.s. (proof based on MGF).

8.3 Weak law of large numbers (WLLN).

8.4 Applications of CLT and WLLN.

9. Finite Markov chains (14L)

9.1 Definition of a sequence of discrete r.v.s. , Markov property, Markov chain, state space and finite Markov chain (M.C.), one step and n step transition probability, stationary transition probability,stochastic matrix P, one step and n step transition probability matrix (t.p.m.)

9.2 Chapman – Kolmogorov equations, t.p.m. of random walk and gambler's ruin problem.

9.3 Applications and real life situations.

Books Recommended

1.Adke, S.R., Manjunath, S.M. (1984). An introduction to finite Markov processes, Wiley Eastern.

2.Arora Sanjay and Bansi Lal (1989). Mathematical Statistics (I^{st} Edition), Satya Prakashan 16/17698 ,New Delhi .

3.Bhat, B.R. (2000). Stochastic models: Analysis and applications, New Age International.

4.Cramer H.: (1962) Mathematical Method of Stastistics , Asia Publishing House ,Mumbai.

5.Gupta S.C and.Kapoor V.K: (2006). Fundamental Mathematical Statistics, Sultan Chand and Sons , 88,Daryaganj , New Delhi.

6.Hogg, R.V.and Craig A.T.(1970). Introduction Mathematical Stastistics (III^{rd} Edition), Macmillan Publishing Company .Inc. New York.

7.Hoel, P. G., Port, S.C. and Stone, C.J. (1972) : Introduction to stochastic processes, Wiley Eastern.

8.Lindgren B.W.: (1976) Statistical Theory (III^{rd} Edition) Collier Macmillan international Edition , Macmillan Publishing Co.Inc. New York.

9.Medhi J. (1982). Stochastic processes, Wiley Eastern

10.Mood. A.M. , Graybill , F.Bose ,D.C.: (1974) Introduction to theory of Statistics. (III^{rd} Edition) Mc- Graw Hill Series.

11.Mukhopdhyay, P (1996). Mathematical Statistics, New Central Book Agency.

12.Rohatgi , V.K. (1975) An Introduction to probability Theory and Mathematical Statistics ,Wiley Eastern Ltd .New Delhi

13.Ross, S. (1996) Stochastic processes, John Wiley.

14. Ross, S. (2000). Introduction to probability models, 7^{th}edn, Academic Press

15.Srinivasan, S.K. and Mehta, K.M. (1981). Stochastic Processes, Tata Mc-Graw Hill.

16.Taylor, H N and Karlin, S. (1984). An introduction to stochastic modelling Academic Press.

STATISTICS (SPECIAL-IV)
Title: PRACTICALS

Sr. No	Title of the Experiment	No. of practicals
1	Model sampling from Cauchy and Laplace distributions	1
2	Fitting of lognormal distribution	1
3.	Constriuction of uniformally most powerful (UMP) test plotting of power function of a test.	1
4	Non-parametric tests: Wilcoxon's signed rank test. Mann-whitney U test, median test and Kolmogorov –Smirnov test.	2
5.	SPRT for Bernoulli, Binormial, Hypergeometric distributions. (graphical respreentation also)	1
6.	SPRT for normal, exponential distribution (graphical respresentation also)	1
7	Analysis of CRD (equal and unequal replications)pairwise comparision of treatments, using critical difference (C.D)Check normality using normal probability plot.	1
8	Analysis of R.B.D. pairwise comparison of treatments using i)C.D. ii)Tukey and Scheffs procedure. Efficiency of RBD w.r.t.CRD	2
9	Analysis of L.S.D., Pairwise comparison of treatments using C.D and box plot, efficiency of LSD w.r.t i) CRD ii) RBD	2
10	Analysis of covariance in CRD, testing β=0.	1
11	Analysis of covariance in RBD, testing β=0.	1
12	Analysis of 2^2 and 2^3 factorial experiments in RBD.	1
13	Analysis of 23 factorial experiments in RBD (partial confounding)	1
14	Analysis of 23 factorial experiments in RBD (total confounding)	1
15.	Linear programming problem I (Simplex Method)	1
16	Linear Programming Problem II (simplex Method)	1
17	Transportation Problem and assignment Problem	2
18	CPM & PERT	2
19	Simulation:1)Simulation from standard probability distributions: a)normal distribution using Box –Muller transformation, b) exponential distribution	1
	Total Practicals	24

Laboratory Equipments: Laboratory should be well equipped with sufficient number of electronic calculators and computers along with necessary software, printers and UPS.

MATHEMATICAL STATISTICS (GENERAL)

STATISTICAL INFERENCE

Note:

1) Mathematical Statistics can be offered only as a general level subject.

2) A student of three Year B.A. Degree Course opting Mathematical Statistics will not be allowed to optr Applied Statistics in any of the three years of the course.

First Term

THEORY OF ESTIMATION

1. Point Estimation **(4L)**

1.1 Notion of a parameter, parameter space , sample space as a set of all possible values of $(X_1, X_2, . . . , X_n)$,general problem of estimating an unknown parameter by point and interval estimation.

1.2 Point Estimation: Definition of an estimator, distinction between estimator and estimate, illustrative examples.

1.3 Mean Square Error (MSE) of an estimator.

2. Methods of Estimation **(10L)**

2.1 **Method of moments :** Derivation of moment estimators for standard distributions. Illustrations of situations where M.L.E. and moment estimators are distinct and their comparison using mean square error.

2.2 Definition of likelihood as a function of unknown parameter, for a random sample from i) discrete distribution ii) continuous distribution, distinction between likelihood function and p.d.f./ p.m.f.

2.3 **Method of maximum likelihood:** Derivation of maximum likelihood estimator (M.L.E.) for parameters of only standard distributions (case of two unknown parameters only for normal distribution). Use of iterative procedure to derive M.L.E.of location parameter μ of Cauchy distribution. Invariance property of M.L.E.

2.4 a) M.L.E. of θ in uniform distribution over i) $(0, \theta)$ ii)$(- \theta, \theta)$ iii) $(m\theta, n\theta)(m<n)$
b) M.L.E. of θ in $f(x ; \theta)= Exp \{-(x- \theta)\}, x > \theta$.c) M.L.E. of location parameter in Laplace distribution.

3. Criteria of Estimation

3.1 **Unbiasedness** **(4L)**

Definition of an unbiased estimator, biased estimator, positive and negative bias, illustrations and examples (these should include unbiased and biased estimators for the same parameters).

Proofs of the following results regarding unbiased estimators.

a) Two distinct unbiased estimators of $\phi(\theta)$ give rise to infinitely many estimators.

b) If T is an unbiased estimator of θ , then ø (T) is unbiased estimator of ø(θ)provided ø (.) is a linear function.

3.2 **Variance of the estimator** **(4L)**

Notion of the Best Linear Unbiased Estimator and uniformly minimum variance unbiased estimator (UMVUE), uniqueness of UMVUE whenever it exists.

3.3 **Sufficiency** **(7L)**

Concept and definition of sufficiency, statement of the Fisher-Neyman factorization theorem with proof for discrete probability distribution. Pitmann – Koopman form and sufficient statistic; Exponential family of probability distributions and sufficient statistic.

Proofs of the following properties of sufficient statistics.

i) If T is sufficient for θ, then ø (T) is also sufficient for θ provided ø is a one to one and onto function.

ii) If T is sufficient for θ then T is also sufficient for ø (θ).

iii) M.L.E. is a function of sufficient statistic.

3.4 **Efficiency** **(7L)**

Fisher information function: Amount of information contained in statistic T = T(X1, X2, . . . , X_n). Statement regarding information in sample and in a sufficient statistic T.

Cramer- Rao Inequality : Statement and proof of Cramer - Rao inequality, Cramer– Rao Lower Bound(CRLB),definition of minimum variance bound unbiased estimator (MVBUE) of ø (θ) . Proofs of following results:

a) If MVBUE exists for θ then MVBUE exists for ø (θ) where ø (.) is a linear function.

b) If T is MVBUE for θ then T is sufficient for θ.

Comparison of variance with CRLB, relative efficiency of T1 w.r.t. T2 for (i) unbiased (ii) biased estimators. Efficiency of an unbiased estimator T w.r.t. CRLB.

3.5 **Asymptotic Behavior of an Estimator** **(6L)**

Chebychev's inequality for discrete and continuous distributions.

Consistency: Definition, proof of the following theorems:

a) An estimator is consistent if its bias and variance both tend to zero as the sample size tends to infinity.

b) If T is consistent estimator of θ and φ (.) is a continuous function, then Φ (T) is a consistent estimator of φ (θ).

4. Interval Estimation **(6L)**

4.1 Notion of interval estimation, definition of confidence interval (C.I), length of C.I., Confidence bounds, confidence coefficient.

4.2 Definition of pivotal quantity and its use in obtaining confidence intervals.

4.3 Interval estimation for the following cases.

i) Mean (μ) of normal distribution (σ2known and σ2 unknown).

ii) Variance (σ2) of normal distribution (μ known and μ unknown).

iii) Median, quartiles using order statistics.

Second Term

TESTING OF HYPOTHESES

5. Parametric Tests **(15 L)**

5.1 (a) Statistical hypotheses, problem of testing of hypotheses. Definition and illustrations of

(1) simple hypothesis, (2) composite hypothesis, (3) test of hypothesis, (4) critical region, (5) type I and type II errors. Probabilities of type I error and type II error. Problems for calculating the probabilities of errors of two kinds.

(b) Definition and illustrations of (i) level of significance, (ii) observed level of significance (p-value), (iii) size of a test, (iv) power of a test.

5.2 Definition of most powerful(M.P.) level α- test of simple null hypothesis against simple alternative. Statement of Neyman - Pearson (N-P) lemma for constructing the most powerful level α- test of simple null hypothesis against simple alternative hypothesis. Illustrations.

5.3 Power function of a test, power curve, definition of uniformly most powerful (UMP) level α -test for one sided alternative. Illustrations.

6. Likelihood ratio tests **(9 L)**

6.1 Notion of likelihood ratio test (LRT), λ (x)=Sup L(θ0|x) / Sup L(θ|x)

6.2 Construction of LRT for H0: θ ε Q against H1: θ Ï Q for the mean of normal distribution for i) known σ^2 ii) unknown σ^2 with one tailed and two tailed hypotheses.

6.3 LRT for variance of normal distribution for i) known μ ii) unknown μ with one tailed and two tailed hypotheses.

6.4 LRT for parameter of binomial, exponential etc. Two tailed alternative hypotheses,

6.5 LRT as a function of sufficient statistics, statement of asymptotic distribution of $-2 \log_e \lambda(x)$.

7. Sequential Tests **(9 L)**

7.1 Sequential test procedure for simple null hypothesis against simple alternative hypothesis and its comparison with fixed sample size N-P test procedure.

7.2 Definition of Wald's SPRT of strength (α, β). Illustration for standard distributions like

Bernoulli, Poisson, Normal and Exponential.

7.3 SPRT as a function of sufficient statistics. Graphical representation of SPRT.

8. Non-parametric Tests **(15 L)**

8.1 Concept of non- parametric tests. Distinction between a parametric and a nonparametric Test.

8.2 Concept of distribution free statistic. One tailed and two tailed test procedure of (i) Sign test, ii) Wilcoxon signed rank test (iii) Mann-Whitney U test, (iii) Run test, one sample and two samples problems

8.3 Empirical distribution function Sn (x). Properties of S_n (x) as estimator of F (.). Kolmogorov - Smirnov test for completely specified univariate distribution (one Sample problem only) for two sided alternative hypotheses. Comparison with chisquare test.

Books Recommended:

1. Agarwal, B.L. (2003). Programmed Statistics, second edition, New Age International Publications , Delhi.
2. Arora, S. and Bansi Lal. (1989) : New Mathematical Statistics, first edition, Satya Prakashan, New Delhi.
3. Daniel, W.W. (2000) Applied Nonparametric Statistics, Duxbury Press Boston.
4. Dudeweitz, E.J. and Mishra, S.N. (1988). Modern Mathematical Statistics, John Wiley and Sons, Inc
5. Gibbons J.D.(1971). Non parametric Statistical Inference, McGraw Hill Book Company, New York.
6. Hoel, P.G.,Port, S. and Stone, C. (1971). Introduction to Statistical Theory, Houghton Mifflin Company (International) Dolphin .
7. Hogg, R.V. and Craig, R.G. (1989). Introduction to Mathematical Statistics (fourth edition,.Collier Macmillan International Edition, Macmillan Publishing Co. Inc., New York.
8. Kale, B.K. and Muralidharan, K. (2015). Parametric Inference: An Introduction. Narosa, New Delhi.9.
9. Kendall, M. and Stuart, A. (1943) The advanced Theory of Statistics, Vol 1, Charles and Company Ltd., London.
10. Lindgren, B.W.(1976). Statistical Theory (third edition), Collier Macmillan International Edition,Macmillan publishing Co., Inc. New York.
11. Kunte, S ., Purohit, S.G. and Wanjale, S.K. : Lecture Notes On Nonparametric Tests.
12. Mood, A.M., Graybill, F. and Bose, D. C .(1974). Introduction to the theory of Statistics (third edition) International Student Edition, McGraw Hill.
13. Ramchandran, K.M. and Tsokos C. P. (2009). Mathematical Statistics with Applications, Academic Press.
14. Rohatgi, V.K. (1976). An introduction to Probability Theory and Mathematical Statistics, Wiley Eastern Ltd., New Delhi.
15. Siegel, S. (1956). Nonparametric methods for the behavioral sciences, International Student Edition, McGraw Hill,New York.

APPLIED STATISTICS (GENERAL)

1) Applied Statistics can be opted only as a General Level subject.
2) A Student of the Three Year B.A. Degree course opting for Applied Statistics will not be allowed to opt Mathematical Statistics and/or Statistics in any of the three years of the course.

First term

1. **Continuous type distributions** **(12L)**

1.1 Definition of continuous type of r.v. through p.d.f., Definition of distribution function of continuous type r.v. Statement of properties of distribution function of continuous type r.v.s

1.2 Normal distribution p. d. f.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$
$$-\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0.$$
$$= 0, \qquad \text{elsewhere}$$

Standard normal distribution, statement of properties of normal distribution, the graph of p.d.f., nature of probability curve. Computation of probabilities.

2. **Tests of significance** **(18L)**

2.1 Notion of a statistic as a function T(X1, X2 ,..., Xn) and its illustrations.
2.2 Sampling distribution of T(X1, X2 ,..., Xn). Notion of standard error of a statistic.
2.3 Notion of hypothesis, critical region, level of significance.
2.4 Tests based on normal distribution for
(i) $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, $\mu < \mu_0$, $\mu > \mu_0$
(ii) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_1$, $\mu_1 < \mu_2$, , $\mu_1 > \mu_2$
(iii) $H_0 : P = P_0$ against $H_1 : P \neq P_0$, $P < P_0$, $P > P_0$
(iii) $H_0 : P_1 = P_2$ against $H_1 : P_1 \neq P_2$,$P_1 < P_2$, $P_1 > P_2$

3. **Tests based on t, chi-square and F distributions** **(18L)**

3.1 t tests for
(i) $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, $\mu < \mu_0$, $\mu > \mu_0$
(ii) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_1$, $\mu_1 < \mu_2$, , $\mu_1 > \mu_2$
(iii) Paired observations

3.2 Tests for $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$, $\sigma^2 < \sigma^2$, $\sigma^2 > \sigma_0^2$

3.3 Chi square test of goodness of fit.
3.4 Chi square test for independence of attributes: Chi square test for independence of 2 x 2 contingency- table (without proof).Yate's correction not expected.
3.5 Tests for $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$, $\sigma_1^2 < \sigma_2^2$, $\sigma_1^2 > \sigma_2^2$

Second Term

4. Analysis of variance techniques **(12L)**

4.1 Concept of analysis of variance.

4.2 One-way and two – way classification: break up of total sum of squares, analysis of variance table, test of hypotheses of (i) equality of several means,(ii) equality of two means.

4.3 Numerical problems.

5. Non - parametric tests **(10L)**

5.1 Distinction between a parametric and non-parametric problem.

5.2 Concept of distribution free statistic.

5.3 One tailed and two tailed test procedure of

(a) Sign test, (b) Wilcoxon's signed rank test.

5.4 Test for randomness.

6. Statistical quality control **(26L)**

6.1 Meaning and purpose of statistical quality control.

6.2 Control chart:

Chance and assignable causes of quality variations, statistical basis of control chart (connection with test of hypothesis is NOT expected). Control limits (3-sigma limits only).

Criteria for judging lack of control:

(i) One or more points outside the control limits and

(ii) Non-random variations within the control limits : such as a run of seven or more points on either side of the control line, presence of trend or cycle.

6.3 Control charts for variables:

Purpose of R-chart and X chart, construction of R-chart, X-chart when standards are not given. Plotting the simple mean and ranges on X and R charts respectively. Necessity for plotting R-chart. Revision of R-chart.Drawing conclusion about state of process. Revision of X chart. Control limits for future production.

6.4 Control chart for fraction defective (p-chart) only for fixed sample size.

Determination of central line, control limits on p-chart, plotting of sample fraction defectives on p-chart. Revision of p-chart, determination of state of control of the process and interpretation of high and low spots on p-chart. Estimation of central line and control limits for future production.

6.5 Control chart for number of defects per unit (c-chart)

Construction of c chart when standards are not given. Plotting of number of defects per unit on c-chart, determination of state of control of the process, revision of control limits for future production.

6.6 Identification of real life situations where these charts can be used.

Note: (i) Proof or derivations of results are not expected.
(ii) Emphasis should be given on numerical problems.

Books Recommended

1.Goon, Gupta and Dasgupta (1968): Fundamental's of Statistics, Vol. I, The World Press Pvt.Ltd. Calcutta.
2.Gupta, S. P.(2014) : Statistical Methods, Sultan Chand and Sons, Delhi.
3.GuptaS.C., and Kapoor V.K.(1994) : Fundamental s of Applied Statistics , Sultan Chand & Sons,Delhi.
4.Hoel, P. G.(1965): Introduction. of Mathematical Statistics, John Wiley and Sons Co. New York.
5.Larson H.J.(1982): Introduction to Probability Theory and Statistical Applications, A Wiley International Edition.
6.Meyer, P. L.(1970): Introductory Probability Theory and Statistical Applications, Addison-Wesley Publishing Company.
7. Walpole*, R.E.* (1982):Introduction to Statistics, Macmillan Publishing Co. New York.

STATISTICAL PRE-REQUISITES (General)

The course Statistical Pre-requisites may be opted only by candidates opting for one of the social sciences as their special subject at the B.A. Degree Examination. The course Statistical Pre-requisites cannot be opted by those who opt for any of the courses in Statistics Groups for their B.A. Examination

First term

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(i) $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, $\mu < \mu_0$, $\mu > \mu_0$

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3.GuptaS.C., and Kapoor V.K.(1994) : Fundamental s of Applied Statistics , Sultan Chand & Sons,Delhi.

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