

SAVITRIBAI PHULE PUNE UNIVERSITY,PUNE

BOARD OF STUDIES IN MATHEMATICS

Syllabus for T.Y.B.A.

Subject: MATHEMATICS

(With effect from June 2015)

Structure of the course:

AMG 3	Real Analysis I + Real Analysis II
MG 3	Group Theory + Ring Theory
MS 3	Metric Spaces + Complex Analysis
MS4	Ordinary Differential equation +Partial differential equation
FMG 3	Financial Mathematics + Graph Theory

Details of Syllabus:

Paper AMG-3: Real Analysis I + Real Analysis II

First Term : Real Analysis I

1. Sets and functions: **[12 Lectures]**

Operations on sets, Functions, Real-valued functions, Equivalence countability, Real numbers, Cantor set, Least upper bounds

2. Sequences of Real Numbers: **[18 Lectures]**

Definition of sequence and subsequence, Limit of a sequence, Convergent sequences, Monotone sequences, Divergent sequences, Limit superior, Limit inferior, Cauchy sequences.

3. Series of Real numbers:

[18 Lectures]

Convergent and divergent series, series with non-negative terms, alternating series, Conditional and Absolute convergence, Rearrangement of series, Tests of absolute convergence, series whose terms form a non-increasing sequence, The class l^2 .

Text book:

R. R. Goldberg, Methods of real analysis, Oxford & I. B. H. Publications, 1970.
Ch. 1, Art 1.1 to 1.7; Ch. 2, Art 2.1 to 2.10; Ch. 3, Art 3.1 to 3.7 and 3.10

Reference Books:

1. Ajit Kumar and S.Kumaresan, A Basic Course in Real Analysis, CRC Press, Second Indian Reprint 2015.
2. D. Somasundaram, B. Choudhary - A first course in Mathematical Analysis, Narosa Publishing House, 1997.
3. Robert, G. Bartle, Donald Sherbert - Introduction to real analysis, Third edition, John Wiley and Sons.
4. Shantinarayan and Mittal - A course of Mathematical Analysis, Revised edition, S. Chand and Co.(2002).
5. S.C. Malik and Savita Arora - Mathematical Analysis , New Age International Publications, third Edition,(2008).

Second Term : Real Analysis II

1. Riemann Integral:

[16 Lectures]

Sets of measure zero, Definition and existence of Riemann integral, properties of Riemann integral, Fundamental theorem of integral calculus, mean value theorems of integral calculus.

2. Improper Integrals:

[16 Lectures]

Definition of improper integral of first kind, comparison test, test, absolute and conditional convergence, integral test for convergence of series, definition of improper integral of second kind, Cauchy principal value.

3. Sequences and series of functions: [16 Lectures]

Point wise and uniform convergence of sequences of functions, consequences of uniform convergence, convergence and uniform convergence of series of functions, integration and differentiation of series of functions.

Text Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and I. B. H. Publication Co., 1970 Ch. 7, Art. 7.1 to 7.4 and 7.8 Ch. 9, Art 9.1 to 9.5
2. First course in mathematical analysis, D somsundaram, B Chuadhari, Narosa Publishing house 2009. Ch. 8, Art 8.5

Reference Books:

1. Ajit Kumar and S.Kumaresan, A Basic Course in Real Analysis, CRC Press, Second Indian Reprint 2015.
2. Robert, G. Bartle, Donald Sherbert - Introduction to real analysis, Third edition, John Wiley and Sons.
3. Shantinarayan and Mittal - A course of Mathematical Analysis, Revised edition, S. Chand and Co.(2002).
4. S.C. Malik and Savita Arora - Mathematical Analysis , New Age International Publications,Third Edition,(2008).

Paper MG 3 : Group Theory + Ring Theory

First term : Group Theory

- 1. Groups:** [8 lectures]
Binary Operations, Isomorphic Binary Structures, Groups.
- 2. Subgroups:** [8 lectures]
Subgroups, Cyclic Groups.
- 3. Permutations:** [16 lectures]
Cosets, Direct Product: Groups of Permutations, Orbits, Cycles, Alternating Groups, Cosets and the Theorem of Lagrange, Direct Products.

4. Homomorphisms and Factor Groups: [16 lectures]
Homomorphisms, Factor Groups, Factor Group Computations and Simple Groups.

Text book:

John B. Fraleigh, A First Course in Abstract Algebra, Seventh Edition, Pearson.
[Articles: Section 2 to Section 6, Section 8 to Section 10, Section 11(only Direct Product), Section 13, Section 14, Section 15]

Second Term : Ring Theory

1. Rings and Fields: [16 Lectures]
Rings and Fields, Integral Domains, The Fields of Quotients of an Integral Domain, Rings of Polynomials, Factorization of Polynomials over a Field.

2. Ideals and Factor Rings: [16 Lectures]
Homomorphisms and Factor Rings, Prime and Maximal Ideals.

3. Factorization: [16 Lectures]
Unique Factorization Domains , Euclidean Domain Euclidean Domains, Gaussian Integers and Multiplicative Norms

Text Book:

John B. Fraleigh, A First Course in Abstract Algebra, Seventh Edition, Pearson.
Articles: Section 18 to Section 23, Section 26, Section 27, Section 45, Section 46, Section 47.

Reference Books:

1. Joseph, A. Gallian, Contemporary Abstract Algebra,(4th Edition), Narosa Publishing House.
2. I.N. Herstein. Abstract Algebra, (3rd Edition), Prentice Hall of India, 1996.
3. N.S. Gopalkrishnan, University of Algebra, Wiley Eastern 1986.
4. C. Musili, Rings and Modules, Narosa Publishing House, 1992.

Paper MS 3: Metric Spaces + Complex Analysis

Term First : Metric Spaces

1. **Introductory Concepts** [14 Lectures]
Definition and examples of metric spaces, open spheres and closed spheres, neighborhoods, open sets, equivalent Metrics, interior points, closed sets, limit points and isolated points, closure of a set, boundary points, distance between sets and diameter of a set, subspace of a metric space, product metric spaces.
2. **Completeness** [8 Lectures]
Convergent sequences, Cauchy sequences, complete spaces, dense sets and nowhere dense sets (only definition)
3. **Continuous Functions:** [6 Lectures]
Definition and characterizations, extension theorem, uniform continuity, homeomorphism
4. **Compactness** [14 Lectures]
Compact spaces, sequential compactness, equivalence of compactness and sequential compactness, compactness and finite intersection property, continuous functions and compact spaces.
5. **Connectedness** [6 Lectures]
Separated sets, disconnected and connected sets.

Text Book:

Metric Spaces (second Edition), Pawan K. Jain,
Khalil Ahmad, Narosa Publishing House. Sections: 2.1 to 2.13, 3.1 to 3.5,
4.1 to 4.4, 5.1 to 5.6, 6.1, 6.2.

Reference Books:

1. Topology of Metric Spaces, S. Kumaresan, Narosa Publishing House
2. First Course in Metric Spaces, B. K. Tyagi, Cambridge University Press
3. Metric Spaces, Satish Shirali, H. Vasudeva, Springer
4. Principles of Mathematical Analysis, W. Rudin.

Second Term : Complex Analysis

1. Complex Numbers **[6 Lectures]**

Sums and products, Basic algebraic properties, Further properties, Vectors and Moduli, Complex Conjugates, Exponential Form, Products and powers in exponential form, Arguments of products and quotients, Roots of complex numbers, Examples, Regions in the complex plane.

2. Analytic functions **[12 Lectures]**

Functions of Complex Variables, Limits, Theorems on limits, Limits involving the point at infinity, Continuity, Derivatives, Differentiation formulas, Cauchy-Riemann Equations, Sufficient Conditions for differentiability, Polar coordinates, Analytic functions, Harmonic functions.

3. Elementary Functions **[7 Lectures]**

The Exponential functions, The Logarithmic function, Branches and derivatives of logarithms, Some identities involving logarithms, Complex exponents, Trigonometric functions, Hyperbolic functions.

4. Integrals **[12 Lectures]**

Derivatives of functions, Definite integrals of functions, Contours, Contour integral, Examples, Upper bounds for Moduli of contour integrals, Anti-derivatives, Examples, Cauchy-Goursat's Theorem (without proof), Simply and multiply Connected domains. Cauchy integral formula, Derivatives of analytic functions. Liouville's Theorem and Fundamental Theorem of Algebra.

5. Series **[5 Lectures]**

Convergence of sequences and series, Taylor's series, Laurent series (without proof), examples.

6. Residues and Poles **[6 Lectures]**

Isolated singular points, Residues, Cauchy residue theorem, residue at infinity, types of isolated singular points, residues at poles, zeros of analytic functions, zeros and poles.

Text Book:

J.W. Brown and R.V. Churchill, Complex Variables and Applications, International Student Edition, 2009. (Eighth Edition).

Chapter 1 : §1 to §11. Chapter 2: §12, §15 to §26. Chapter 3 : §29 to §35. Chapter 4 : §37 to §46 and §48 to §53. Chapter 5 : §55 to §60 and §62. Chapter 6: §68 to §76.

Reference Books:

1. S. Ponnusamy, Complex Analysis, Second Edition (Narosa).
2. S. Lang, Complex Analysis, (Springer Verlag).
3. A.R. Shastri, An Introduction to Complex Analysis, (MacMillan).

Paper MS4 : Ordinary Differential Equations + Partial differential equation

First Term: Ordinary Differential Equations

1. Linear Differential Equations with constant coefficients: [12 lectures]

The auxiliary equations. Distinct roots, repeated roots, Complex roots, particular solution. The operator $1/f(D)$ and its evaluation for the functions x^m , e^{ax} , $e^{ax}v$ & xv and the operator $1/(D^2 + a^2)$ acting on $\sin ax$ and $\cos ax$ with proofs.

2. Non-Homogeneous Differential Equations: [14 lectures]

Method of undetermined coefficients, Method of variation of parameters, Method of reduction of order, The use of a known solution to find another.

3. Power series solutions: [12 lectures]

Introduction and review of power series, Linear equations and power series, Convergence of power series, Ordinary points and regular singular points.

4. System of First-Order Equations: [10 lectures]

Introductory remarks, linear systems, homogeneous linear systems with constant Coefficients, Distinct roots, repeated roots, Complex roots.

Text Books:

Elementary Differential Equations, Rainville and Bedient, Macmillan Publication .

Reference Books:

1. Ordinary and Partial Differential Equation, by M.D.Raisinghania, S.Chand and Company LTD, 2009
2. Differential Equations by George F. Simmons, Steven G. Krantz, Tata McGraw-Hill.
3. W.R. Derrick and S.I. Grossman, A First Course in Differential Equations with applications. CBS Publishers and distributors, Delhi-110 032. Third Edition.
4. Shanti Narayan, Integral Calculus, S. Chand and Company.
5. Daniel Murray, Introductory Course in Differential Equations, Orient Longman

Second Term : Partial differential equation

1. Ordinary Differential Equations in More Than Two Variables

[22 Lectures]

- (a) Surface and Curves in Three Dimensions
- (b) Simultaneous Differential Equations of the First Order and the First Degree in Three Variables.
- (c) Methods of solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$
- (d) Orthogonal Trajectories of a System of curves on a Surface.
- (e) Pfaffian Differential Forms and Equations.
- (f) Solution of Pfaffian Differential Equations in Three Variables

2. First Order Partial Differential Equations:

[26 lectures]

- (a) Genesis of First Order Partial Differential Equations.
- (b) Classification of Integrals.
- (c) Linear Equations of the First Order.
- (d) Pfaffian Differential Equations.
- (e) Compatible Systems.
- (f) Charpit's Method.
- (g) Jacobi's Method.
- (h) Integral Surfaces through a given curve.
- (i) Quasi-Linear Equations.

Text Books:

1. Ian Sneddon, Element of Partial Differential Equations, McGraw-Hill Book Company, McGraw-Hill Book Company. Chapter 1: §1 to §6.
2. T. Amaranath, An Elementary Course in Partial Differential Equations, Narosa Publishing, House 2nd Edition, 2003 (Reprint, 2006). Chapter 1: §1 to §10.

Reference Books:

1. Frank Ayres Jr., Differential Equations, McGraw-Hill Book Company, SI Edition (International Edition, 1972)
2. Ravi P. Agarwal and Donal O'Regan, Ordinary and Partial Differential Equations, Springer, First Edition (2009).
3. W.E. Williams, Partial Differential Equations, Clarendon Press, Oxford,(1980).
4. K. Sankara Rao, Introduction to Partial Differential Equations, Third Edition, PHI.

FMG 3: Financial Mathematics + Graph Theory

First Term: Financial Mathematics

- 1. Mathematical models in economics** [06 Lectures]
Introduction, a model of the market, market equilibrium and excise tax.
- 2. The elements of finance and the cobweb model:** [10 Lectures]
Interest and capital growth, income generation, the interval of compounding, stability of market equilibrium, the general linear case and economic interpretation.
- 3. Introduction to optimization:** [8 Lectures]
Profit maximization, critical points, optimization in an interval and infinite intervals.
- 4. The derivative in economics:** [12 Lectures]
Elasticity of demand, profit maximization again, competition versus monopoly, the efficient small firm, startup and breakeven points.
- 5. Linear equations :** [4 Lectures]
Making money with matrices, a two-industry 'economy', arbitrage portfolios and state prices, IS-LM analysis.
- 6. The input-output model:** [8 Lectures]
An economy with many industries and the technology matrix.

Text Book

Martin Anthony and Norman Biggs, Mathematics for Economics and Finance Methods and Modelling, Cambridge University Press, Reprint 2012.

Reference Book

Edward T. Dowling , Mathematical Economics, Second Edition, Schaum's Outline Series, McGraw Hill International Edition.

Second Term : Graph Theory

1. An Introduction to Graphs Lectures]

[18

The definition of a Graph, Graphs and Models, More Definitions, Vertex Degree, Sub graphs, Paths and Cycles, The Matrix Representation of Graphs, Fusion

2. Trees and Connectivity Lectures]

[14

Definition and Simple Properties, Bridges, Spanning Trees, Connector Problems, Shortest Path Problems, Cut Vertices and Connectivity.

3. Euler Tours and Hamiltonian Cycles

[8 Lectures]

Euler Tours, The Chinese Postman Problem, Hamiltonian Graphs, The Travelling Salesman Problem.

4. Directed Graphs

[8 Lectures]

Definitions, In degree and Out degree, Tournaments, Traffic Flow.

Text Book

A First Look at Graph Theory, John Clark and Derek Allan Holton, Allied Publishers Ltd.(1991), Chapter No. 1,2,3 and 7.

Reference Books

1. Introduction to Graph Theory, R. J. Wilson, Pearson(2003)
2. Graph Theory, Hararay, Narosa Publishers, New Delhi(1989)
3. Graph Theory, Narsing Deo, Prentice Hall of India Pvt. Ltd.(1987)
4. Basic Graph Theory, K. R. Parthsarathy, TataMcGraw Hill Publisher Co. Ltd.