# SAVITRIBAI PHULE PUNE UNIVERSITY, PUNE <br> Board of Studies in Mathematics 

Syllabus for S. Y. B. A

Subject: MATHEMATICS
(With effect from June 2020)

## Preamble:

Savitribai Phule Pune University has decided to change the syllabi of various faculties from June,2019. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects board of studies in mathematics with concern of teachers of mathematics from different colleges affiliated to Savitribai Phule Pune University has prepared the syllabus of S. Y. B. A. Mathematics. To develop the syllabus the U.G.C. Model curriculum is followed.


#### Abstract

Aims: (i) Give the students a sufficient knowledge of fundamental principles, methods and a clear perception of innumerous power of mathematical ideas and tools and know how to use them by modeling ,solving and interpreting. (ii) Reflecting the broad nature of the subject and developing mathematical tools for continuing further study in various fields of science and technology. (iii) Enhancing students' overall development and to equip them with mathematical modeling abilities, problem solving skills, creative talent and power of communication necessary for various kinds of employment . (iv) Enabling students to develop a positive attitude towards mathematics as an interesting and valuable subject of study.


## Objectives:

(i) A student should be able to recall basic facts about mathematics and should be able to display knowledge of conventions such as notations, terminology and recognize basic geometrical figures and graphical displays, state important facts resulting from their studies.
(ii) A student should get a relational understanding of mathematical concepts and concerned structures, and should be able to follow the patterns involved, mathematical reasoning.
(iii) A student should get adequate exposure to global and local concerns that explore them many aspects of Mathematical Sciences.
(iv) A student be able to apply their skills and knowledge ,that is, translate information presented verbally into mathematical form, select and use appropriate mathematical formulae or techniques in order to process the information and draw the relevant conclusion.
(v) A student should be made aware of history of mathematics and hence of its past, present and future role as part of our culture.

## Course Outcome:

| Sr. No | Course Outcomes |
| :--- | :--- |
| $\mathbf{1}$ | On completion of this course, students should be able to understand basic functions <br> and the fundamentals of calculus required in financial mathematics |
| 2 | To understand the concepts of basic and linear algebra and applications of <br> mathematical methods to the problem of economics. |
| 3 | To graph and differentiate the simple functions |
| 4 | To be able calculate basic quantities in financial mathematics and to apply these <br> concepts in financial markets and real-life situations |

## Structure of the course:

|  | Semester - III |  | Semester | -IV |
| :--- | :--- | :--- | :--- | :--- |
| Paper I | MG-3 | Graph Theory | MG-4 | Linear Algebra |
| Paper II | AMG-3 | Calculus of Several <br> Variables | AMG-4 | Vector Calculus |
| Paper III | FMG-3 | Operational Research | FMG-4 | Optimization <br> Techniques |
| Paper IV | MS-1 | Problem Course based <br> on MG-3 and AMG-3 | MS-3 | Problem Course based <br> on MG-4 and AMG-4 |
| Paper IV | MS-2 | Number Theory | MS-4 | Computational <br> Geometry |

All five above courses are compulsory.

Equivalence of Previous syllabus along with new syllabus:

|  | Old course | New Course |
| :--- | :--- | :--- |
| Paper I | MG-2 : Discrete <br> Mathematics + Linear <br> Algebra | MG-3: Graph Theory <br> and <br> MG-4 : Linear Algebra |
| Paper II | AMG-2 : <br> Multivariable Calculus-I <br> Multivariable Calculus-II | AMG-3: Calculus of Several Variable <br> and <br> AMG-4 : Vector Calculus |
| Paper III | FMG-2 : Operational <br> Research + Optimization <br> Techniques | FMG-3: Operational Research <br> and <br> FMG-4: Optimization Techniques |
| Paper IV | MS-1: Problem Course <br> based on MG-2 and AMG-2 | MS-1:Problem Course based on MG- <br> 3 and AMG-3 <br> and <br> MS-3: Problem Course based on MG- <br> 4 and AMG-4 |
| Paper V | MS-2: Number Theory + <br> Computational Geometry | MS-2: Number Theory <br> and <br> MS-4: Computational Geometry |

## Details of Syllabus:

Note: The number of credits for each course will be as per the rules and regulations for all B. A courses. The number of lectures will vary accordingly.

If a course is of $\mathbf{3}$ credits then number of lectures for the course is $\mathbf{4 8}$ lectures. If a course is of 2 credits then number of lectures for the course is $\mathbf{3 6}$ lectures.

# Semester - III <br> <br> MG-3 : Graph Theory 

 <br> <br> MG-3 : Graph Theory}

## Unit 1. Introduction

[04 Lectures]
1.1 What is a Graph?
1.2 Application of Graphs
1.3 Finite and Infinite Graphs
1.4 Incidence and Degree
1.5 Isolated Vertex, Pendant Vertex and Null Graph

## Unit 2. Paths and Circuits

[12 Lectures]
2.1 Isomorphism
2.2 Subgraphs
2.4 Walks, Paths, and Circuits
2.5 Connected Graphs, Disconnected Graphs, and Components
2.6 Euler Graphs
2.7 Operations on Graphs
2.8 More on Euler Graphs
2.9 Hamiltonian Paths and Circuits
2.10 The Traveling Salesman Problem

Unit 3. Trees and Fundamental Circuits
[14 Lectures]
3.1 Trees
3.2 Some Properties of Trees
3.3 Pendant Vertices in a Tree
3.4 Distance and Centers in a Tree
3.5 Rooted and Binary Trees
3.6 On Counting Trees
3.7 Spanning Trees
3.8 Fundamental Circuits
3.10 Spanning Trees in a Weighted Graph

Unit 4. Cut-Sets and Cut-Vertices
[06 Lectures]
4.1 Cut-Sets
4.2 Some Properties of a Cut-Set
4.3 All Cut-Sets in a Graph
4.4 Fundamental Circuits and Cut-Sets
4.5 Connectivity and Separability

## Text Book :

1. Narsingh Deo, "Graph Theory with Applications to Engineering and Computer Science" Printice-Hall, of India Pvt. Lt. New Delhi.

Unit 1 : Chapter 1: Sec.1.1 to 1.5
Unit 2: Chapter 2: Sec. 2.1 to 2.10 (Excluding 2.3)
Unit 3: Chapter 3: Sec. 3.1 to 3.10 (Excluding 3.9)
Unit 4: Chapter 4 : Sec. 4.1 to 4.5

## Reference books:

1. John Clark and Derek Holton, A First Look at Graph Theory (Allied Publishers)
2. Robin J. Wilson, Introduction to Graph Theory, Fourth Edition(low price edition)
3. Introduction to Graph Theory, Douglas West 2nd edition.
4. A Textbook of Graph Theory, Balakrishnan, R., Ranganathan, K.

## AMG-3 : Calculus of Several Variables

Unit-1 Limits and Continuity
[06 lectures]
1.1 Functions of Several Variables :- Functions of two variables, Domain and Range, Graphs, Level Curves, Functions of Three or More Variables
1.2 Limits and Continuity.

## Unit-2 Partial Derivatives and Differentiability

[10 lectures]
2.1 Definition and examples.
2.2 Higher Derivatives, Clairaut's Theorem (Statement Only), Partial Differential Equations, Wave equation.
2.3 Differentiable function, Differentials
2.4 Chain Rule, Homogeneous Functions, Euler's theorem

## Unit-3 Extreme Values

[08 lectures]
3.1 Extreme values of functions of two variables.
3.2 Necessary conditions for extreme values.
3.3 Second Derivative Test (without proof).
3.4 Lagrange Multipliers ( with one constraints)

## Unit-4 Multiple Integrals

[12 lectures]
4.1 Iterated Integrals, Fubini's Theorem (Statement only)
4.2 Double integral over general regions, Change of order of integration for two variables.
4.3 Double integral in Polar coordinates.
4.4 Triple integrals, Evaluation of triple integrals. Triple integrals in spherical coordinates
4.5 Jacobians, Change of variables in multiple integrals .(Results without proofs)

Text book: Multivariable Calculus 7th Edition By James Stewart, Brooks/Cole, Cengage Learning, 2012, 2008.

Unit 1:- Chapter 14: Sec- 14.1, 14.2
Unit 2:- Chapter 14: Sec- 14.3(except the Cobb-Douglas production function), 4.4
(except Tangent Planes and Linear Approximations), Sec-14.5
Unit 3:- Chapter 14: Sec 14.7, 14.8 (except two constraints)
Unit 4:- Chapter 15: Sec 15.2, 15.3, 15.4, 15.7 (without Riemann sum and
Application), 15.9, 15.10

## Reference Books:

1. Basic Multivariable Calculus, J. E. Marsden, A. J. Tromba, A. Weinstein, Springer

Verlag (Indian Edition).
2. Shanti Narayan, R.K. Mittal, A Text-book of Vector Calculus, S.Chand and Company.
3. D.V. Widder, Advanced Calculus (2nd Edition), Prentice Hall of India ,NewDelhi,(1944).
4. T.M. Apostol, Calculus Vol. II (2nd Edition), John Wiley, New York, (1967).

## FMG-3 : Operational Research

Unit 1. Modeling with Linear Programming:
[6 lectures]
1.1 Two variable LP Model
1.2 Graphical LP solution
1.3 Selected LP Applications
1.4 Graphical Sensitivity analysis.

Unit 2. The Simplex Method:
[10 lectures]
2.1 LP Model in equation form
2.2 Transition from graphical to algebraic solutions
2.3 The simplex method
2.4 Artificial starting solutions.

Unit 3. Duality:
[4 lectures]
3.1 Definition of the dual problem
3.2 Primal dual relationship.

Unit 4. Transportation Model:
[10 Lectures]
4.1 Definition of the Transportation model
4.2 The Transportation algorithm.

Unit 5. The Assignment Model:
[06 Lectures]
5.1 The Hungarian method
5.2 Simplex explanation of the Hungarian method.

## Text Book:

Hamdy A. Taha, Operation Research (Eighth Edition, 2009), Prentice Hall of India Pvt. Ltd, New Delhi.
Ch.2: 2.1,2.2,2.3(2.3.4, 2.3.5, 2.3.6). Ch.3: 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 (3.6.1).
Ch.4: 4.1, 4.2. Ch.5: 5.1,5.3 (5.3.1, 5.3.2, 5.3.3), 5.4(5.4.1, 5.4.2).
Reference Books:

1. Frederick S. Hillier, Gerald J. Lieberman, Introduction to Operation Research (Eighth Edition) Tata McGraw Hill.
2. J K Sharma, Operations Research (Theory and Applications, second edition, 2006), Macmilan India Ltd.
3. Hira and Gupta, Operation Research.

MS-1 : Problem Course Based on MG-3 and AMG-3

MS-2 : Number Theory

## Unit 1. Divisibility :

[06 Lectures]
1.1 Introduction
1.2 Divisibility
1.3 Primes

Unit 2. Congruences
[08 Lectures]
2.1 Congruences
2.2 Solution of Congruences
2.3 The Chinese Remainder Theorem

Unit 3. Greatest integer function:
[08 Lectures]
3.1 Greatest integer function
3.2 Arithmetic functions
3.3 The Mobius Inversion formula

Unit 4. Quadratic Reciprocity:
[08 Lectures]
4.1 Quadratic residues
4.2 Quadratic reciprocity
4.3 The Jacobi Symbol

Unit 5. Diophantine Equations :
[06 Lectures]
5.1 Diophantine equations $\mathrm{ax}+\mathrm{by}=\mathrm{c}$
5.2 Pythagorean triplets.

## Text Book:

1. I. Niven, H. Zuckerman and H.L. Montgomery, An Introduction to Theory of Numbers, 5th Edition, John Wiley and Sons.
(§1.1-§1.3, §2.1- §2.3, §3.1- §3.3, §4.1-§4.3, §5.1 and §5.3.)
Reference Book:
2. David M. Burton, Elementary Number Theory (Second Ed.), Universal Book Stall, New Delhi, 1991.

## Semester - IV

## MG-4 : Linear Algebra

Unit-1: Matrices and System of Linear Equations
[06 lectures]
1.1 Row echelon form of a matrix, reduced row echelon form of a matrix.
1.2 Definition of rank of a matrix using row echelon or row reduced echelon form.
1.3 System of linear equations- Introduction, matrix form of linear system, definition of row equivalent matrices.
1.4 Consistency of homogeneous and non-homogeneous system of linear equations using rank, condition for consistency.
1.5 Solution of System of Equations: Gauss elimination and Gauss-Jordan elimination method, examples.

Unit-2: Vector Spaces-I
[10 lectures]
2.1 Definition and Examples.
2.2 Subspaces.
2.3 Linear Dependence and Independence.
2.4 Basis of Vector Space

Unit-3: Vector Spaces-II
[08 lectures]
3.1 Dimension of a Vector Space.
3.2 Row, Column and Null Space of a matrix.
3.3 Rank and nullity.

Unit-4: Linear Transformations
[12 lectures]
4.1 Definition and Examples, Properties, Equality.
4.2 Kernel and range of a linear Transformation
4.3 Rank-Nullity theorem.
4.4 Composite and Inverse Transformation.
4.5 Matrices and Linear Transformation.
4.6 Basic Matrix Transformations in $\mathrm{R}^{2}$ and $\mathrm{R}^{3}$
4.7 Linear Isomorphism.

## Text Book:

## Howard Anton, Chris Rorres, Elementary Linear Algebra, Application Version, Ninth Edition, Wiley, $11^{\text {th }}$ edition.

Unit-1: Chapter-1: Sec. 1.1, 1.2.
Unit-2: Chapter- Sec. 4: 4.1 to 4.4.
Unit-3: Chapter- Sec. 4: 4.5, 4.7, 4.8
Unit- 4: Chapter- Sec.8: 8.1 to $8.4,1.8,4.9$.

## Reference Books:

(1) K. Hoffman and R. Kunze, Linear Algebra, $2^{\text {nd }}$ edition(2014), Prentice Hall of India, New Delhi
(2) Steven J. Leon, Linear Algebra with Applications, $4^{\text {th }}$ edition(1994), Prentice Hall of India. New Delhi
(3) Vivek Sahai, Vikas Bist, Linear Algebra, $4^{\text {th }}$ Reprint 2017, Narosa Publishing House, New Delhi
(4) Promode Kumar Saikia, Linear Algebra, 2009, Pearson, Delhi
(5) S. Lang, Introduction to Linear Algebra, $2^{\text {nd }}$ edition, 1986, Springer-Verlag, New York, Inc.

## AMG-4 : Vector Calculus

## Unit 1: Vector-Valued Functions

[08 lectures]
1.1 Curves in Space, Limits and Continuity, Derivatives and Motion, Differentiation Rules for Vector Function, Vector Functions of Constant Length.
1.2 Integrals of Vector Functions.
1.3 Arc Length along a Space Curve, Speed on a Smooth Curve, Unit Tangent Vector.
1.4 Curvature of a Plane Curve, Circle of Curvature for Plane Curves, Curvature and Normal Vectors for a Space Curve.
Unit 2: Integrals
[12 Lectures]
2.1 Line Integral of Scalar Functions, Additivity, Line integral in the Plane.
2.2 Vector Fields, Gradient Fields, Line Integral of Vector Fields, Line Integrals with respect to $\mathrm{dx}, \mathrm{dy}, \mathrm{dz}$.
2.3 Work done by a Force over a Curve in Space, Flow Integrals and Circulation for Velocity Fields, Flow across the Simple Closed Plane Curve.
2.4 Path Independence, Conservative and Potential Functions.
2.5 Divergence, Two forms for Green's Theorem, Green's Theorem in the Plane (Proof for special regions),

## Unit 3: Surface Integrals

[08 Lectures]
3.1 Parameterizations of Surfaces, Implicit surfaces.
3.2 Surface integrals, Orientation of Surfaces.
3.3 Surface Integrals of Vector Fields.

## Unit 4: Applications of Integrals

[08 Lectures]
4.1 The Curl Vector Field, Stokes' Theorem(without proof), Conservative Fields and Stokes’ Theorem.
4.2 Divergence in three Dimensions, Divergence Theorem (without proof).
4.3 Unifying the Integral Theorems.

## Text Book:

Thomas' Calculus (14 ${ }^{\text {th }}$ Edition) by Hass, Heil, Weir, Pearson Indian Education Services Pvt. Ltd.
Unit 1: Chapter 13: Sec- 13.1, 13.2, 13.3, 13.4
Unit 2: Chapter 16: Sec-16.1, 16.2, 16.3, 16.4
Unit 3: Chapter 16: Sec- 16.5, 16.6
Unit 4: Chapter 16: Sec- 16.7, 16.8

## Reference books:

(1) Basic Multivariable Calculus byJ.E.Mardson, A.J.Tromba, A. Weinstein, Sppriger Verlag (Indian Edition)
(2) Advanced Calculus by M.R. Spiegel, Schaum Series.
(3) Advanced Calculus (IInd Edition) byD.V. Widder, Prentice Hall of India, New Delhi(1944).
(4) Advanced Calculus by John M. H. Olmsted, Eurasia Publishing House, New Delhi(1970)
(5) Calculus Vol. II (IInd Edition) by T.M. Apostol, John Wiley, New York (1967).

## Unit 1. Network Models

[09 Lectures]
CPM and PERT, Network representation, Critical Path Computations, Construction of the time schedule, Linear programming formulation of CPM, PERT calculations.

Unit 2. Decision Analysis and Games
[09 Lectures]
Decision under uncertainty, Game theory, Some basic terminologies, Optimal solution of two person zero sum game, Solution of mixed strategy games, graphical solution of games, linear programming solution of games.

Unit 3. Replacement and Maintenance Models
[06 Lectures]
Introduction, Types of failure, Replacement of items whose efficiency deteriorates with time.

## Unit 4. Sequencing Problems

[06 Lectures]
Introduction, Notation, terminology and assumptions, processing n jobs through two machines, processing n jobs through three machines.

Unit 5. Classical Optimization Theory
[06 Lectures]
Unconstrained problems, Necessary and sufficient conditions, Newton Raphson method, Constrained problems, Equality constraints (Lagrangian Method Only).

## Text Book:

1. Hamdy A. Taha, Operation Research (Eighth Edition, 2009), Prentice Hall of India Pvt. Ltd, New Delhi. Ch.6: 6.5 (6.5.1 to 6.5.5).
Ch.13: 13.3, 13.4(13.4.1,13.4.2,13.4.3). Ch.18: 18.1(18.1.1, 18.1.2), 18.2 (18.2.1).
2. J K Sharma, Operations Research (Theory and Applications, second edition, 2006), Macmilan India Ltd. Ch.17: 17.1,17.2, 17.3.
Ch.20: 20.1, 20.2, 20.3, 20.4.
Reference Books:
3. Frederick S. Hillier, Gerald J. Lieberman, Introduction to Operation Research (Eighth Edition) Tata McGraw Hill.
4. Hira and Gupta, Operation Research.

MS-3: Problem Course Based on MG-4 and AMG-4

MS-4 : Computational Geometry

Unit 1. Two dimensional Transformations

Introduction, Representation of Points, Transformations and Matrices, Transformation of Points, Transformation of Straight Lines, Midpoint Transformation, Transformation of Parallel Lines, Transformation of Intersecting Lines, Rotation, Reflection, Scaling, Combined Transformations, Transformation of the Unit Square, Solid Body Transformation, Translations and Homogeneous Coordinates, Rotation About an Arbitrary Point, Reflection Through an Arbitrary Line,
Projection - A Geometric Interpretation of Homogeneous Coordinates, Overall Scaling, Points at Infinity.

## Unit 2. Three Dimensional Transformations:

[10 Lectures]
Three Dimensional Scaling and Shearing, Three Dimensional Rotation. Three Dimensional Reflection. Three Dimensional Translation. Multiple Transformations, Rotations about an Axis Parallel to a coordinate axis, Rotation about an Arbitrary Axis in Space, Reflection Through an Arbitrary Plane. Affine and Perspective Geometry, Orthographic Projections, Axonometric Projections, Oblique Projections, Perspective Transformations. Techniques for generating perspective views, Vanishing points.

## Unit 3. Plane Curves

[10 Lectures]
Curve representation, non-parametric curves, parametric curves, parametric representation of a circle, parametric representation of an Ellipse, parametric representation of a parabola, parametric representation of a Hyperbola.

Unit 4. Space Curves Beizer curves:
[06 Lectures]
Introduction, definition, properties (without proofs), curve fitting (up to $n=3$ ), equation of the curve in matrix form (up to $n=3$ ).

## Text-Book:

D.F. Rogers, J. Alan Adams, Mathematical Elements of Computer Graphics, Second Edition, McGraw-Hill Publishing Company.
( $\$ 2.2$ to $2.20,3.1$ to $3.15,3.17,4.1$ to $4.8,5.8$ )

