SYBA
SEMESTER -III

Subject: Statistics (Special-I)

ST-23853 : CONTINUOUS PROBABILITY DISTRIBUTIONS

Unit. 1 Continuous Univariate Distributions: (10)

1.1 Continuous sample space: Definition, illustrations.

1.2 Continuous random variable: Definition, probability density function (p.d.f.), cumulative distribution function (c.d.f.), properties of c.d.f. (without proof), probabilities of events related to random variable.

1.3 Expectation of continuous r.v., expectation of function of r.v. $E[g(X)]$, mean, variance, geometric mean, harmonic mean, raw and central moments, skewness, kurtosis, mean deviation about mean.

1.4 Moment generating function (MGF): Definition, properties. Cumulant generating function (CGF): Definition, properties.

1.5 Mode, quartiles($Q_{1,} Q_{2,} Q_3$)

1.6 Probability distribution of function of r. v. : $Y = g(X)$ using i) Jacobian of transformation for $g(.)$ monotonic function and one-to-one, on to functions,

ii) Distribution function for $Y = X^2$, $Y = |X|$ etc., iii) M.G.F. of $g(X)$.

Unit. 2 Continuous Bivariate Distributions: (12)

2.1 Continuous bivariate random vector or variable (X, Y): Joint p. d. f., joint c. d. f, properties (without proof), probabilities of events related to r.v. (events in terms of regions bounded by regular curves, circles, straight lines). Marginal and conditional distributions. Independence of r.v.s X & Y and also its extension to k r.v.s.

2.2 Expectation of of function of r.v. $E[g(X, Y)]$, joint moments, $Cov(X, Y), Corr\ (X, Y),$ conditional mean, conditional variance, $E[E(X|Y = y)] = E(X)$ *and* $E[E(Y/X = x) = E(Y),$ regression as a conditional expectation.

2.3 Theorems on expectation:

i) $E(X + Y) = E(X) + E(Y)$, $(ii)\ E(XY) = E(X)\ E(Y), if\ X\ and\ Y$ are independent, generalization to k variables. $E(aX + bY + c), Var\ (aX + bY + c)$ (statement only proof not expected).

2.4 M.G.F.:$M_{X,Y}\ (t_1, t_2)$, properties, M.G.F. of marginal distribution of r.v.s., properties

i) $M_{X,Y}\ (t_1, t_2) = M_X\ (t_1, 0)\ M_Y\ (0, t_2)$ if X and Y are independent r. v.s.,

ii) $M_{X+Y}\ (t) = M_{X,Y}\ (t, t)$

iii) $M_{X+Y} = M_X\ (t)\ M_Y\ (t)$ if X and Y are independent r.v.s.

2.5 Probability distribution of transformation of bivariate r. v. $U = \phi_1\ (X,Y)$, $V = \phi_2(X,Y)$.

Unit.3 Uniform or Rectangular Distribution: **(06)**

3.1 Probability density function (p.d.f.)

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0 & , otherwise \end{cases}$$

Notation : $X \sim U[a, b]$.

3.2 Sketch of p. d. f. , c. d. f. and sketch of c.d.f., mean, variance, symmetry, MGF.

3.3 Distribution of i) $\frac{X-a}{b-a}$,ii) $\frac{b-X}{b-a}$,iii) $Y = F(X)$, where $F(X)$ is the c. d. f. of continuous r.v. X. Application of the result to model sampling. (Distributions of $X + Y, X - Y, XY$ and X/Y are not expected.)

Unit.4 Normal Distribution: **(14)**

4.1Probability density function (p.d.f.):

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} & ; -\infty < X < \infty, -\infty < \mu < \infty, \ \sigma > 0 \\ 0 & ; \qquad otherwise \end{cases}$$

Notation: $X \sim N\ (\mu\ \sigma^2)$.

4.2 p. d. f. curve, identification of scale and location parameters, nature of probability curve, mean, variance, MGF, CGF, central moments, cumulants, skewness, kurtosis, mode, quartiles(Q_1,Q_2,Q_3), points of inflexion of probability curve, mean deviation, additive property.

4.3 Probability distribution of : i) $\frac{X-\mu}{\sigma}$,standard normal variable (S.N.V.), ii) $aX + b$, iii) $aX + bY + c$, where X and Y are independent normal variates. Probability distribution of $\bar{X}$, the mean of n $i.i.d.$ $N\ (\mu, \sigma^2)$ r. v s.,

4.4 Computations of normal probabilities using normal probability integral tables. Central limit theorem (CLT) for $i.i.d.$ r.v.s. with finite positive variance(statement only), its illustration for Poisson and Binomial distributions.

Unit.5 Exponential Distribution: **(06)**

5.1Probability density function (p. d. f.):

$$f(x) = \begin{cases} \alpha e^{-\alpha x} ; & x \ge 0 , \alpha > 0 \\ 0 & ; \ otherwise \end{cases}$$

Notation : $X \sim Exp(\alpha)$.

5.2 Nature of density curve, interpretation of α as a scale and $\frac{1}{\alpha}$ as mean, mean, variance, MGF, CGF, skewness and kurtosis.

5.3 c.d.f., graph of c.d.f., lack of memory property, quartiles(Q_1,Q_2,Q_3), mean deviation about mean, additive property.

5.4 Distribution of $min(X, Y)$ and $max(X, Y)$ with X, Y $i.i.d.$ exponential r.v.s.

References:

1. A.Sanjay and Bansilal(1989) : New Mathematical Statistics(First Edition), Satya Prakashan, 16/7698 New Market, New Delhi – 5.
2. A.M.Goon,M.K. Gupta and B. Dasgupta(1986) : Fundamentals of Statistics,Vol. 2,World Press, Calcutta.
3. A.M.Mood,F.A. Graybill and F.A.Boes(1974) : Introduction to Theory of Statistics (Third Edition), McGraw – Hill Series G A 276.17
4. R.V.Hogg and A.T. Craig: Introduction to Mathematical Statistics (Third Edition), Macmillan Publishing Co. Inc. 866, Third Avenue, New York 10022.
5. S.C.Gupta and V.K. Kapoor : Fundamentals of Mathematical Statistic, Sultan Chand and Sons, 23, Daryaganj, New Delhi 110002.
6. S.P.Gupta : Statistical Methods, Sultan Chand and Sons, 23, Daryaganj, New Delhi 110002.
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SYBA
SEMESTER -IV

Subject: Statistics (Special-I)
ST-23854: SAMPLING DISTRIBUTIONS AND INFERENCE

Unit.1 Gamma Distribution: (6)

1.11Probability density function (p. d. f.):

$$f(x) = \begin{cases} \frac{\alpha\lambda}{\Gamma\lambda} x^{\lambda-1} e^{-\alpha x} & ; x > 0,\ \alpha, \lambda > 0 \\ 0 & ;\ otherwise \end{cases}$$

= 0 , otherwise.

Notation: $X \sim G(\alpha, \lambda)$,

1.2 Nature of probability curve, special cases: i) $\alpha = 1$, ii) $\lambda = 1$,MGF, CGF, moments, cumulants, skewness, kurtosis, mode, additive property.

1.3 Distribution of sum of n i.i.d. exponential variables. Relation between distribution function of Poisson and Gamma variates.

Unit. 2 Chi-square Distribution: (10)

2.1 Definition as a sum of squares of i.i.d. standard normal variables. Derivation of the p.d.f. of Chi-square variable with n degrees of freedom (d.f.) using MGF technique.

2.2 Mean, variance, MGF, CGF, central moments skewness, kurtosis, mode, additive property. Use of chi-square tables for calculations of probabilities. Normal approximation (statement only)

2.3 Distribution of $\frac{nS^2}{\sigma^2} = \frac{1}{\sigma^2}\sum_{i=i}^{n}(X_i - \bar{X})^2$for a sample from a normal distribution using orthogonal transformation, independence of $\bar{X}$ and S^2.

Unit. 3 Student's t −distribution: (8)

3.1Definition of T r.v. with n d.f. in the form of $T = \frac{U}{\sqrt{\frac{V}{n}}}$, where $U \sim N\ (0, 1)$ and V is chi-square with n d.f., where U & V are independent random variables.

Notation: $T \sim t_n$

3.2 Derivation of the p.d.f of t distribution, nature of probability curve, mean, variance, moments, mode.

3.3Use of t-tables for calculations of probabilities, statement of normal approximation.

Unit.4 Snedecore's F −distribution: (08)

4.1Definition of F r.v. with n_1 and n_2 d.f. as $F_{n_1,n_2} = \frac{\chi^2_{n_1}/n_1}{\chi^2_{n_2}/n_2}$ Independent chi-square variables with n_1 and n_2 d.f.

Notation: $X \sim F_{n_1,n_2}$

4.2 Derivation of the p.d.f, nature of probability curve, mean, variance, moments, mode.

4.3 Distribution of $\frac{1}{F_{n_1,n_2}}$,use of F −tables for calculation of probabilities.

4.4 Interrelationship between Chi-square, t and F distributions

Unit.5 Test of Hypothesis: (16)

5.1Tests based on chi-square distribution:

a) Test for independence of two attributes arranged in $r \times s$ contingency table, Mc Nemar's test (to be covered in practical only).

b) Test for goodness of fit. (to be covered in practical only)

c) Test for variance against one-sided and two-sided alternatives i) for known mean , ii) for unknown mean.

5.2 Tests based on t −distribution:

a) Tests for population means:

i) one sample with unknown variance and two sample for unknown equal variances tests for one-sided and two-sided alternatives.

ii)$100(1-\alpha)\%$ two sided confidence interval for population mean and difference of means of two independent normal populations.

b) Paired t-test for one-sided and two-sided alternatives.

5.3 Test based on F distribution:

Test for H_0: $\sigma_1^2 = \sigma_2^2$ against one-sided and two-sided alternatives when i) means are known and ii) means are unknown.

References:

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4. M.B.Kulkarni ,S. B. Ghatpande, S. B. and S.D.Gore(1999) : Common Statistical Tests Satyajeet Prakashan, Pune 411029.

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