Savitribai Phule Pune University

(Formerly University of Pune)

Faculty of Science & Technology

F.Y.B.A. (Mathematical Statistics)

Choice Based Credit System Syllabus

To be implemented from Academic Year 2019-2020

Title of the program: B.A. (Mathematical Statistics)

1) Preamble to the syllabus:

The word ***Statistics*** is used in different ways in different contexts. To a cricket fan, Statistics is the information about runs scored or wickets taken by a player. To the manager of a manufacturing unit, Statistics may be the information about the process control. To a medical researcher investigating the effects of a new drug, Statistics are evidence of research efforts. For college student, Statistics are the grades or marks scored in a course. Thus, in all these illustrations Statistics word refers to quantitative data in the area under study. Statistics as a subject is an important branch of knowledge and is devoted to various techniques of collection, presentation, analysis and interpretation of data. It is a science of learning fromdata.

Statistics provides tools for making decisions when conditions of uncertainty prevail. Hence these tools and techniques are used in almost all fields. Statistics is indispensable for people working in fields like agriculture, business, management, economics, finance, insurance, education, biotechnology and medical science etc. Since last two decades, with the help of computers large amount of data can be handled and more sophisticated statistical techniques can be used in an effective manner. Knowledge of different aspects of Statistics has become crucial. There is a continuous demand for statisticians in every field – education, industry, software and research. The syllabus of the three Year B. A. degree course in Statistics is framed in such a way that the students at the end of the course can apply judiciously the statistical tools to a variety of data sets to arrive at someconclusions.

Statistics can be divided into two broad categories, (1) exploratory statistics or descriptive statistics, which is concerned with summarizing data and describing these data, and (2) confirmatory statistics or inferential statistics, which is concerned with making decisions about the population based on thesample.

Up to higher secondary school, students are mostly exposed to descriptive statistics. At the first year a student can take any one of the four subjects related statistics, such as Statistics, Applied Statistics, Mathematical Statistics and Statistical Prerequisites. If the student continues with these subjects at the second year and third year, it is expected that at the end of the degree course a student is able to apply the statistical tools to real life data.

Introduction:

B. A. degree program is three years of duration, with semester pattern for the second and third year and annual examination pattern for the firstyear.The structure of Bachelor of Arts

(B. A.) is as follows.

The student joining the First Year B.A. Course has to take six subjects from 13 groups. The student cannot take more than one subject from one group. There are four subjects related to statistics. These are Statistics (Group L), Applied Statistics (Group L), Mathematical Statistics (Group J) and Statistical Prerequisites (Group K).

Structure of the Subject:

Structure of the subject for first and subsequent three years and the pattern of examination and question papers are as specified below.

Structure of F. Y. B. A. Statistics/Mathematical Statistics/Applied Statistics/ Statistical-Prerequisites

Semester	Subject	Subject code	Title	Credit	Marks
I	Statistics	ST- 13871	Descriptive Statistics I	3	100
	Mathematical Statistics	ST- 13271	Discrete Probability and Probability Distributions	3	100
	Applied Statistics	ST-14171	Descriptive Statistics I	3	100
	Statistical Pre-requisites	ST-13571	Descriptive Statistics I	3	100
II	Statistics	ST- 13872	Descriptive Statistics II	3	100
	Mathematical Statistics	ST- 13272	Discrete Probability Distributions	3	100
	Applied Statistics	ST-14172	Descriptive Statistics II	3	100
	Statistical Pre-requisites	ST-13572	Descriptive Statistics II	3	100

Detailed Syllabus:

Proposed syllabus for the above mentioned course to be implemented from theAcademic Year2019-20 onwards.

It is advised to offer Statistics and Mathematical Statistics to those students who are being interested to study Statistics as special subject at SYBA and TYBA level.

Objectives:

The main objective of this course is to introduce to the students the basic concepts of probability, axiomatic theory of probability, concept of random variable, probability distribution (univariate and bivariate) discrete random variables, expectation and moments of probability distribution.By the end of the course students are expected to be able:

1. to distinguish between random and non-random experiments.
2. to find the probabilities of events.
3. to obtain a probability distribution of random variable (one or two dimensional) in the given situation, and
4. to apply the standard discrete probability distribution to different real life situations.

Pre requisite: Permutation and Combination theory, Binomial theorem, Algebra of sets.

SEMESTER – I

ST- 13271:Discrete Probability and Probability Distributions

1. Basics of Probability:
(10L)

1.1Experiments/Models, Ideas of deterministic and non-deterministic models. Random Experiment, concept of statisticalregularity.

1.2Definitions of - (i) Sample space, (ii) Discrete sample space: finite and countably infinite, (iii) Event, (iv) Elementary event, (v) Complement of an event. (vi) Certain event (vii) Impossibleevent

1.3Concept of occurrence of anevent.Algebra of events. Representation ofoccurrence of following eventsin set theory notations:

(i) at least one of the givenevents,
(ii) none of the givenevents,
(iii) all of the givenevents,
(iv) mutually exclusiveevents,
(v) mutually exhaustiveevents,
(vi) exactly one event out of the givenevents.

1.4Classical definition of probability and itslimitations.Probability model, probability of an event, equiprobable and non-equiprobable sample space,1.5Axiomatic definition ofprobability.Definition of conditional probability of anevent.Definition of independence of two events $P(A \cap B) = P(A) \cdot P(B)$

Pairwise independence and mutual independence for threeevents

Multiplication theorem $P(A \cap B) = P(A)*P(B|A)$. Generalization to $P(A \cap B \cap C)$.

2.Bayes'Theorem:

(4L)

2.1Partition of the samplespace, prior and posterior probabilities.Proof of Bayes' theorem. 2.2Applications of Bayes' theorem in reallife.

3.Univariate Probability Distributions (Defined on Discrete Sample Space):

(6L)

3.1Concept and definition of a discrete randomvariable.Probability mass function (p.m.f.) and cumulative distribution function (c.d.f.), F(·) of discrete random variable, properties ofc.d.f..

3.2Mode and median of a univariate discrete probabilitydistribution.

4.Mathematical Expectation (Univariate Random Variable):

(10L)

4.1Definition of expectation (Mean) of a discrete random variable, expectation of a function of a random variable, m.g.f. and c.g.f. Properties of m.g.f andc.g.f. (with proof)

4.2Definitions of variance, standard deviation (s.d.) and Coefficient of variation

(c.v.) of univariate probability distribution, effect of change of origin and scale on mean, variance ands.d.(only statement)

4.3Definition of raw, central and factorial raw moments of univariate probability Distributions and their interrelations **(**withoutproof).

4.4Coefficients of skewness and kurtosis based onmoments.

5.Some Standard Discrete Probability Distribution:

(18L)

5.1Degenerate distribution (one point distribution), P(x=c)=1, mean and variance,

5.2Uniform discrete distribution on integers 1 to *n*: p.m.f.,

$$P(x) = \frac{1}{n} \quad ;x = 1,2,3,4,\ldots\ldots n.$$

$$= 0 \quad ;\text{otherwise.}$$

Notation: $X \to U(n)$.

Mean, variance, real life situations, comments of mode and median.

5.3 Bernoulli Distribution: p.m.f., mean, variance, moments, distribution of sum of independent identically distributed Bernoulli variables.

5.4 Binomial Distribution: p.m.f.

$$P(x) = \binom{n}{x} p^x q^{n-x} \quad ; \quad x = 0,1,2,3,\ldots\ldots n.$$

$$0 < p < 1,\ q = 1 - p$$

$$= 0 \quad ;\text{otherwise.}$$

Notation: $X \to B(n, p)$.

Computation of probabilities of different events, Recurrence relation for successive probabilities, computation of mode of the distribution,mean, variance, moments, skewness (comments when $p = 0.5$, $p > 0.5$, $p < 0.5$).

5.5 Hypergeometric Distribution :

p.m.f. of the distribution

$$P(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} \quad ;x=0,1,2,3,\ldots\ldots \min(n, M).$$

$$= 0 \quad ;\text{otherwise.}$$

Notation : $X \rightarrow H(N, M, n)$.

Computation of probability, situations where this distribution is applicable, binomial approximation to hypergeometric probabilities, mean and variance of the distribution.

Recommended Books:

1. Agarwal B. L. (2003). Programmed Statistics, second edition, New Age International Publishers, NewDelhi.
2. Gupta, S.C. and Kapoor, V. K. (1983). Fundamentals ofMathematical Statistics, Eighth Edition, Sultan Chand and Sons Publishers, NewDelhi.
3. Hoel P. G. (1971). Introduction to Mathematical Statistics, John Wiley and Sons, New York.
4. Hogg,R.V.andCraigR.G.(1989).IntroductiontoMathematicalStatistics,Ed. MacMillan Publishing Co., NewYork.
5. Mayer,P.(1972).IntroductoryProbability and StatisticalApplications, Addison Wesley Publishing Co.,London.
6. Mood, A. M. and Graybill, F. A. and Boes D.C. (1974). Introduction to the Theory of Statistics, Ed. 3, McGraw Hill BookCompany.
7. Ross S. (2002). A First Course in Probability, Sixth Edition, Pearson Education, Inc. & Dorling Kindersley Publishing,Inc.

SEMESTER – II

ST- 13272: Discrete Probability Distributions

1.Bivariate DiscreteProbabilityDistribution:
(16L)

1.1Definition of two-dimensional discrete random variable, its joint p.m.f. and its distribution function(c.d.f.) with properties. concept of identically distributedr. vs. Computation of probabilities of events in bivariate probabilitydistribution.

1.2Concepts of marginal and conditional probabilitydistributions.Additive property of binomial variables, conditional distribution of X given $X + Y$, where X and Y are independent, $B(n_1, p)$ *and* $B(n_2, p)$ variables.

1.3Independence of two discrete random variables based on joint and marginal p.m.fs

2.Mathematical Expectation (BivariateRandomVariable):
(10L)

2.1Definition of raw and central moments,theorems on expectations of sum and product of two jointly distributed random variables.

2.2Conditionalexpectation.Definitions of conditional mean and conditionalvariance.

2.3Definitionofcovariance,coefficientofcorrelation (ρ),only statement for properties of correlation of coefficient (ρ)*:* i) $-1 \le \rho \le 1$ ii) effect of changeof origin and scale. Independenceanduncorrelatedness of twovariables.

2.4Variance of linear combination of variables Var(aX +bY).

3.Poisson Distributions:
(12L)

3.1p.m.f. of the distribution

$$P(x) = \frac{e^{-m} m^{x}}{x!} \quad ; x = 0,1,2,3,..... \quad m > 0$$
$$= 0 \quad ; otherwise$$

Notation : $X \rightarrow P(m)$

3.2Mean, variance, m.g.f. ,c.g.f.,cumulantsand moments. Skewness and Kurtosis of distribution.

3.3Additive property for Poissondistribution.Conditional distribution of X given (X+Y) for Poissondistribution.Situations where this distribution is applicable.

4. Geometric distribution:
(10L)

4.1Geometric distribution over the range 0, 1, 2, ... with p.m.f.

$P(x) = pq^x$

Geometric distribution over the range 1, 2,3, ... with p.m.f.

$P(x) = pq^{x-1}$.

$0 < p < 1, q = 1 - p.$

Notation: $X \to G(p)$

4.2Mean, variance, m.g.f. and c.g.f. Lack of memory property.

4.3Situations where this distribution is applicable.

Reference Books

1. Hogg, R. V. and Craig R. G. : Introduction to Mathematical Statistics,Ed. 4. (1989), MacMillan Publishing Co., New York.
2. Hoel, P. G. : Introduction to Mathematical Statistics (1962), John Wileyand Sons, New York.
3. Feller, W. : Introduction to Probability Theory and Its Applications, Vol.I (1963), Asian Publishing House, Bombay.
4. Mood, A. M. and Graybill, F. A. and Boes D.C. E. : Introduction to Theoryof Statistics, Ed. 3 (1974), McGraw Hill and Kagakusha Ltd. London.
5. Mayer, P. N. : Introduction to Probability and Statistical Applications,Addison Wesley Publishing Co., Massachusetts).
6. Gupta and Kapoor : Fundamentals of Mathematical Statistics, SultanChand and Sons, New Delhi.
7. Ross : Probability theory, Pearson Publishers.
8. M. B. Kulkarni and S. B. Ghatpande : Discrete Probability and ProbabilityDistributions, SIPF Academy, Nashik.
9. B. L. Agarwal : Programmed Statistics, New Age International Publishers,New Delhi.
10. K. V. S. Sarma : Statistics Made Simple : Do it yourself on PC. PrenticeHall, New Delhi.

Reference Websites :

1. www.stats.unipune.ernet.in (100 Data sets for Statistics Educagtion by Dr Anil P.Gore, Dr. Mrs. S. A. Paranjpe and Madhav B. Kulkarni available in ISPS folder).
2. www.freestatistics.tk
3. www.psychstat.smsu.edu/sbk00.htm
4. www.bmj.bmjournals.com/collections/statsbk/index.shtml
5. www.statweb.calpoly.edu/bchance/stat-stuff.html
6. www.amstat.org/publications/jse/jse-data-archive.html
7. www.statpages.org (Webpages that perform statistical calculations)
8. www.amstat.org/publications/chance (Chance magazine)
9. www.statsci.org/datasets.html (Data sets)
10. www.math.uah.edu/stat (Virtual laboratories in Statistics)
11. www.amstat.org/publications/stats (STATS : the magazine for students of Statistics)
12. www.stat.ucla.edu/cases (Case studies in Statistics)

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